ASU Sound & Vibration Lab

Faculty of Engineering
Ain Shams University

Acoustics

Signal Analysis & Measurement Techniques

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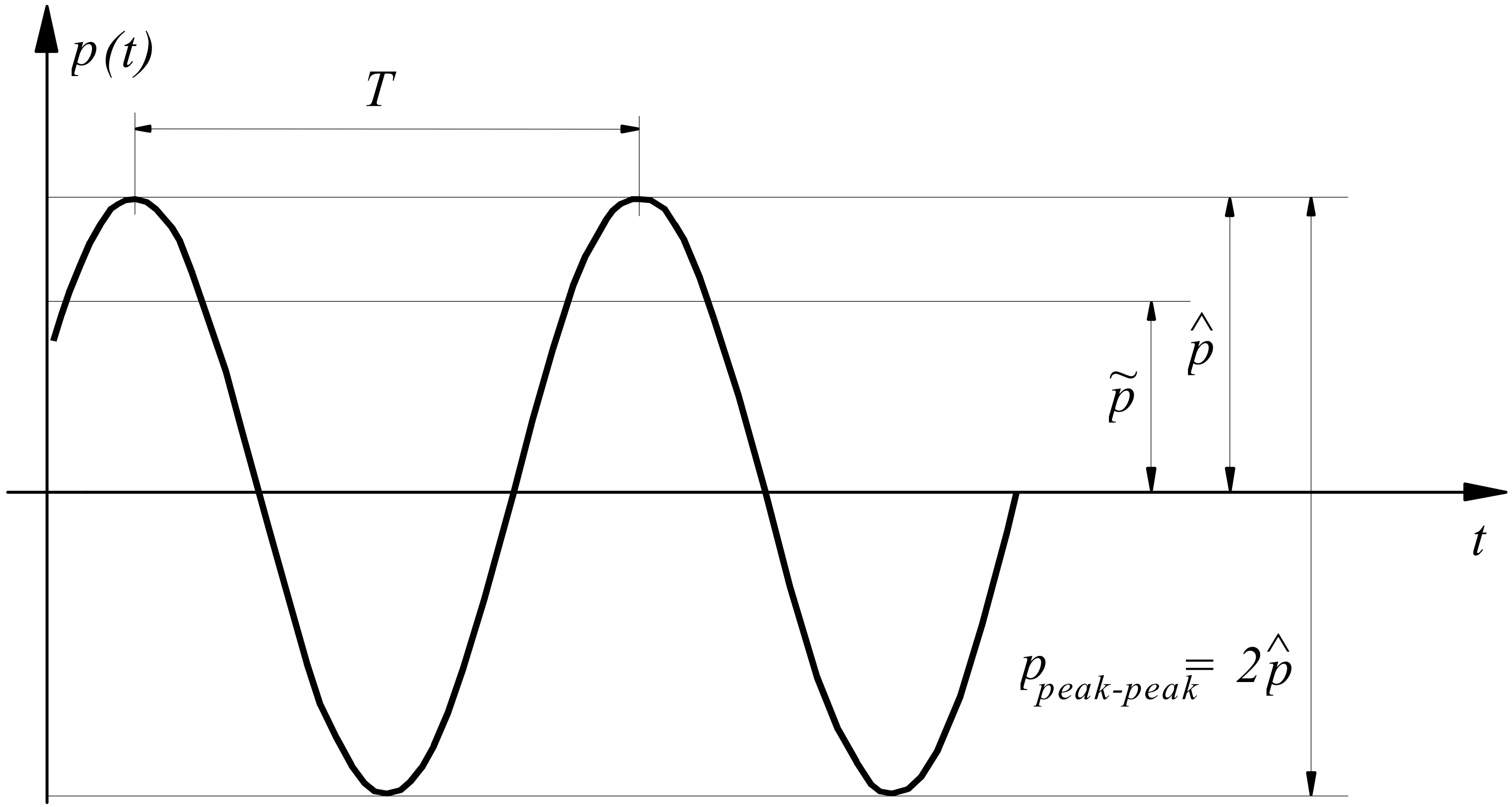
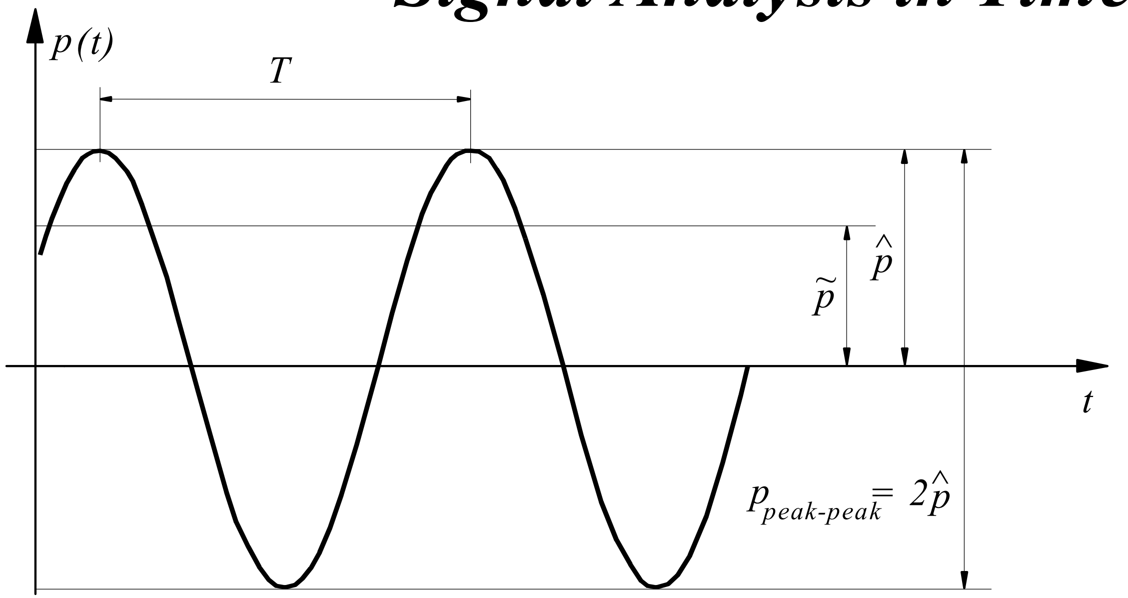
#2: Signal Analysis and Filters

Signal Analysis

Amplitude and Frequency



Signal Analysis in Time Domain



$$f = \frac{1}{T} (\text{Hz})$$

$$\omega = 2\pi f = \frac{2\pi}{T} (\text{rad/s})$$

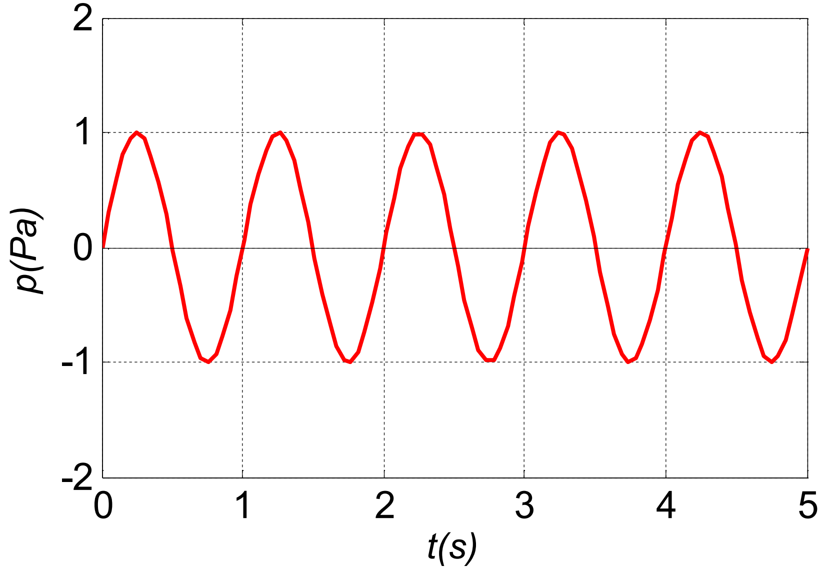
$$p(t) = \hat{p} \sin(\omega t + \phi)$$

Signal Analysis in Time Domain (cont'd)

$$\hat{p} = 1\,\mathrm{Pa}$$

$$f = 1\,\mathrm{Hz}$$

$$p(t) = 1 \times \sin(2\pi t)$$



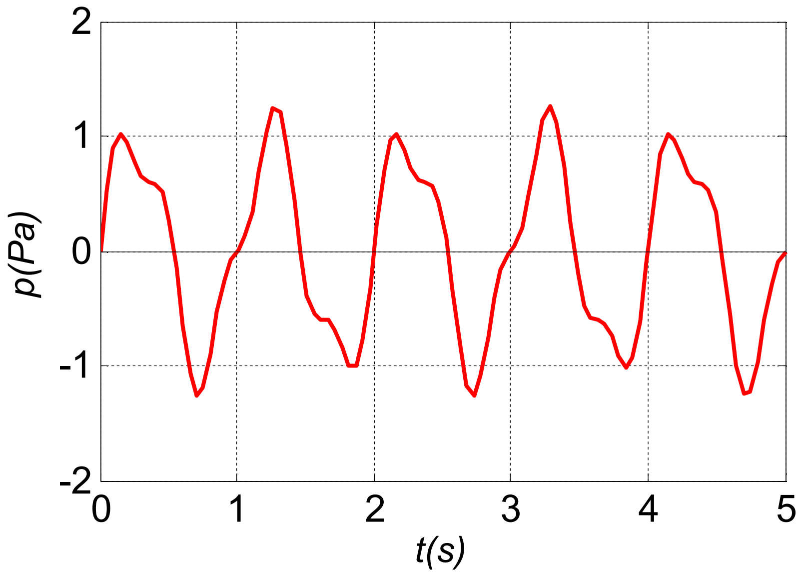
Signal Analysis in Time Domain (cont'd)

$$p(t) = \hat{p}_1 \sin(2\pi f_1 t) + \hat{p}_2 \sin(2\pi f_2 t)$$

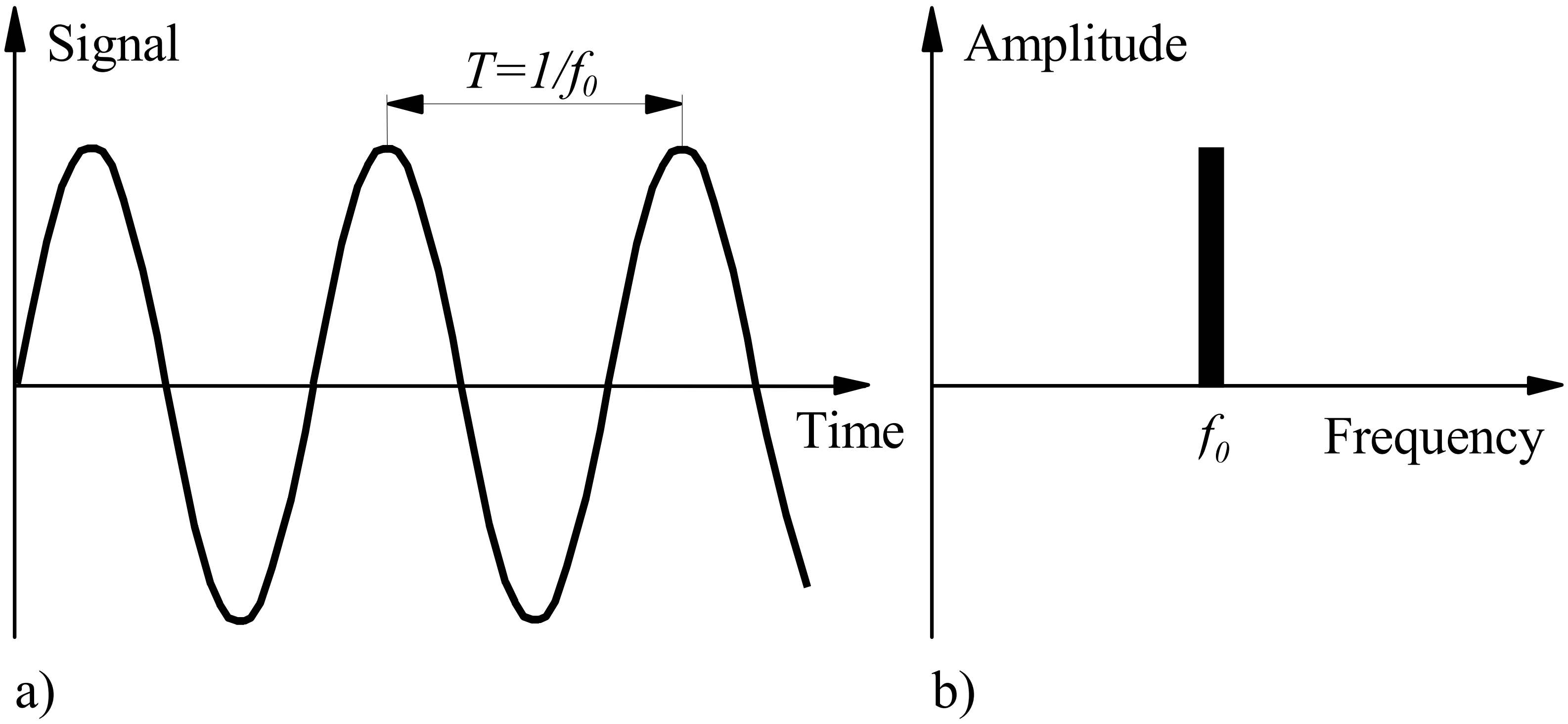
$$\hat{p}_1 = 1\,\text{Pa}\,, \quad \hat{p}_2 = 0.3\,\text{Pa}$$

$$f_1 = 1\,\text{Hz}\,, \quad f_2 = 2.5\,\text{Hz}$$

$$p(t) = 1 \times \sin(2\pi t) + 0.3 \times \sin(5\pi t)$$



Time and Frequency Domain


Signal
$T=1/f_0$
Time
Amplitude
f_0
Frequency
a)
b)

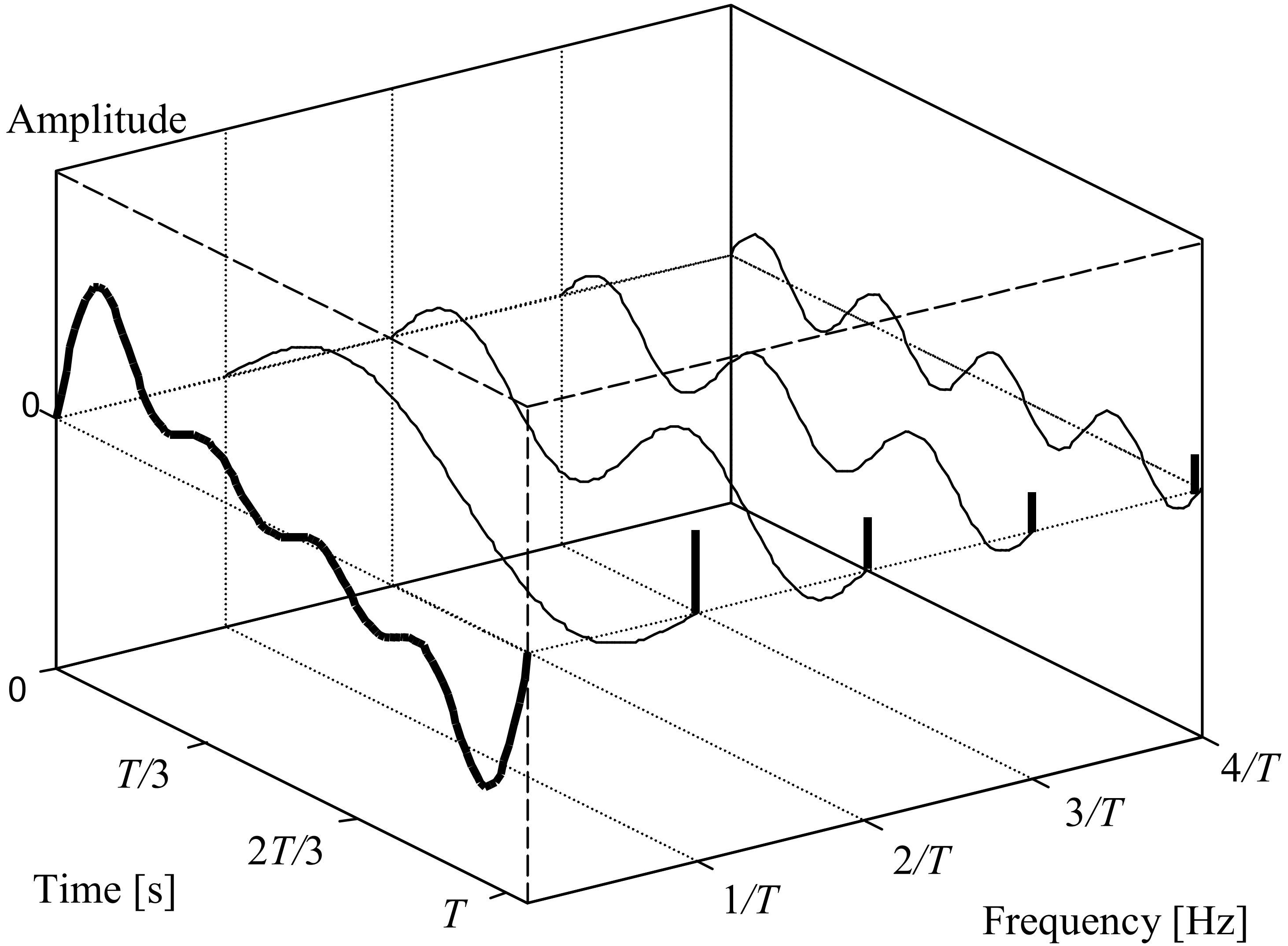
Fourier Analysis

Any Periodic Signal can be considered a sum of a number of harmonic components

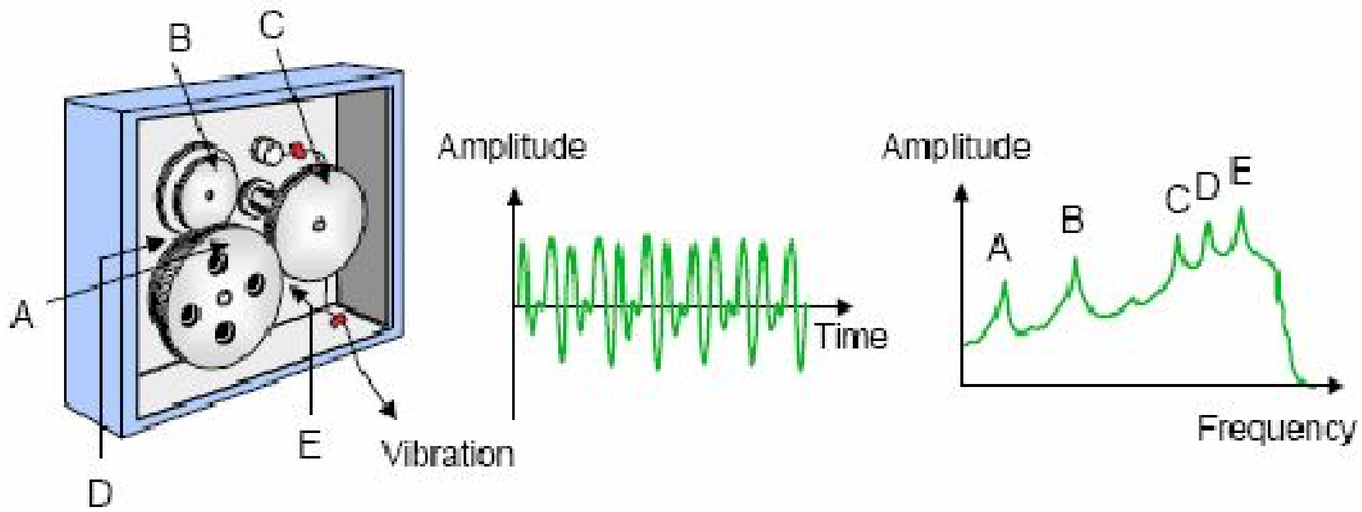
Periodic Signals are very common in Real life

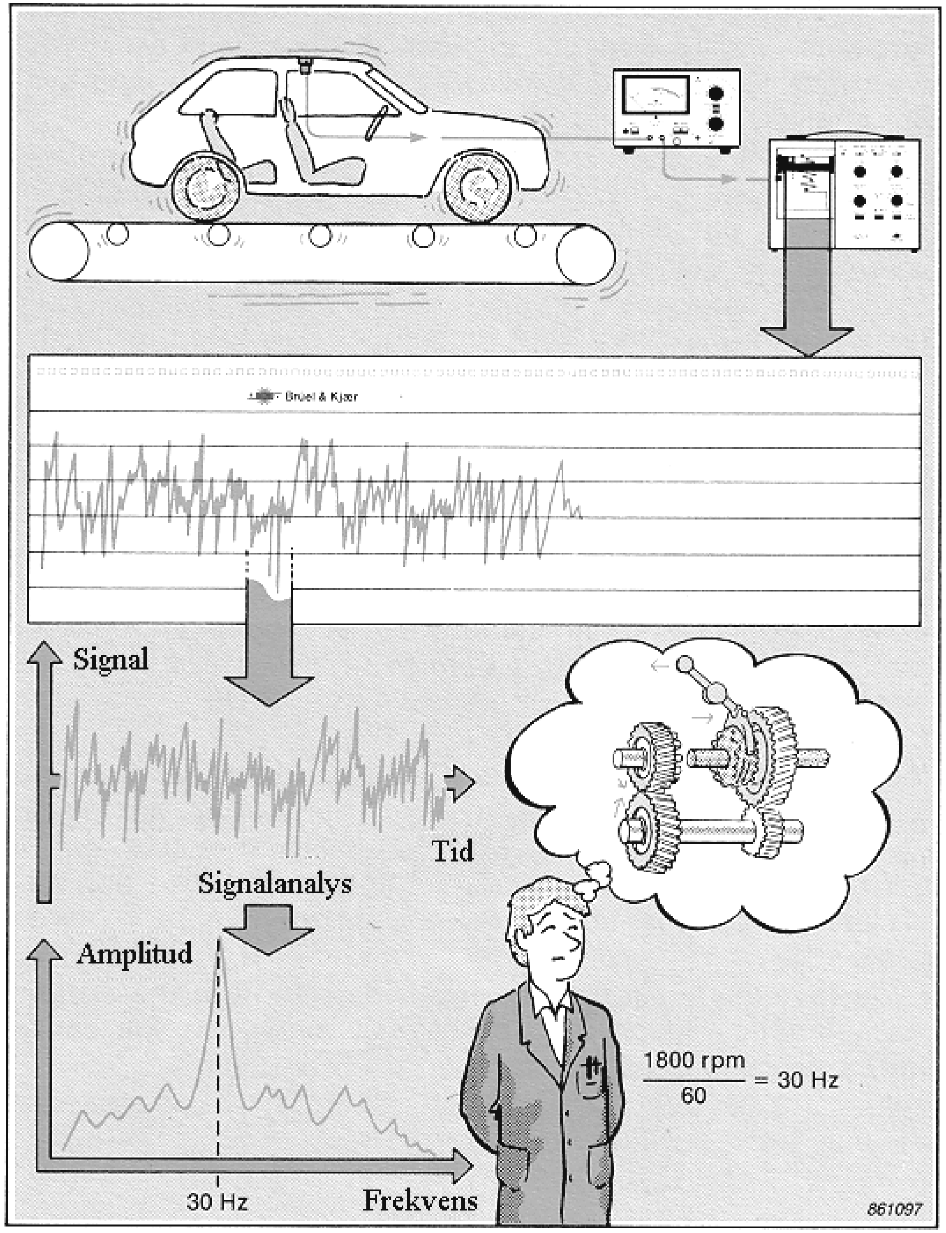
$$p(t) = p(t + nT)$$

$$p(t) = \sum_{n=1}^{N} \hat{p}_n \cos(2\pi\, n f_0 t + \varphi_n)$$


Amplitude
0
0
T/3
2T/3
T
Time [s]
1/T
2/T
3/T
4/T
Frequency [Hz]

Why Make a Frequency Analysis




Brüel & Kjær
Signal
Tid
Signalanalys
Amplitud
30 Hz
Frekvens
1800 rpm / 60 = 30 Hz
861097

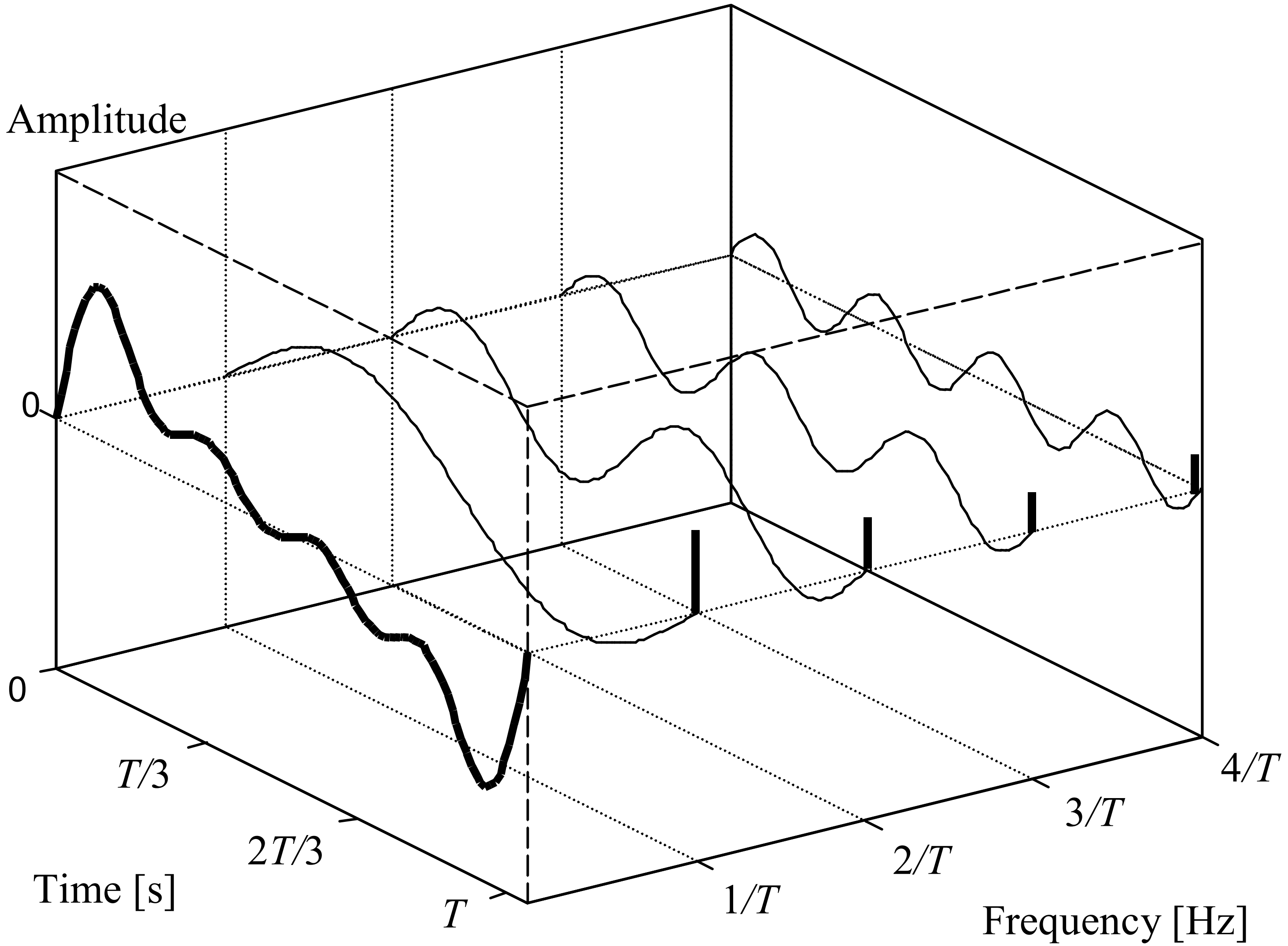
Fourier Analysis

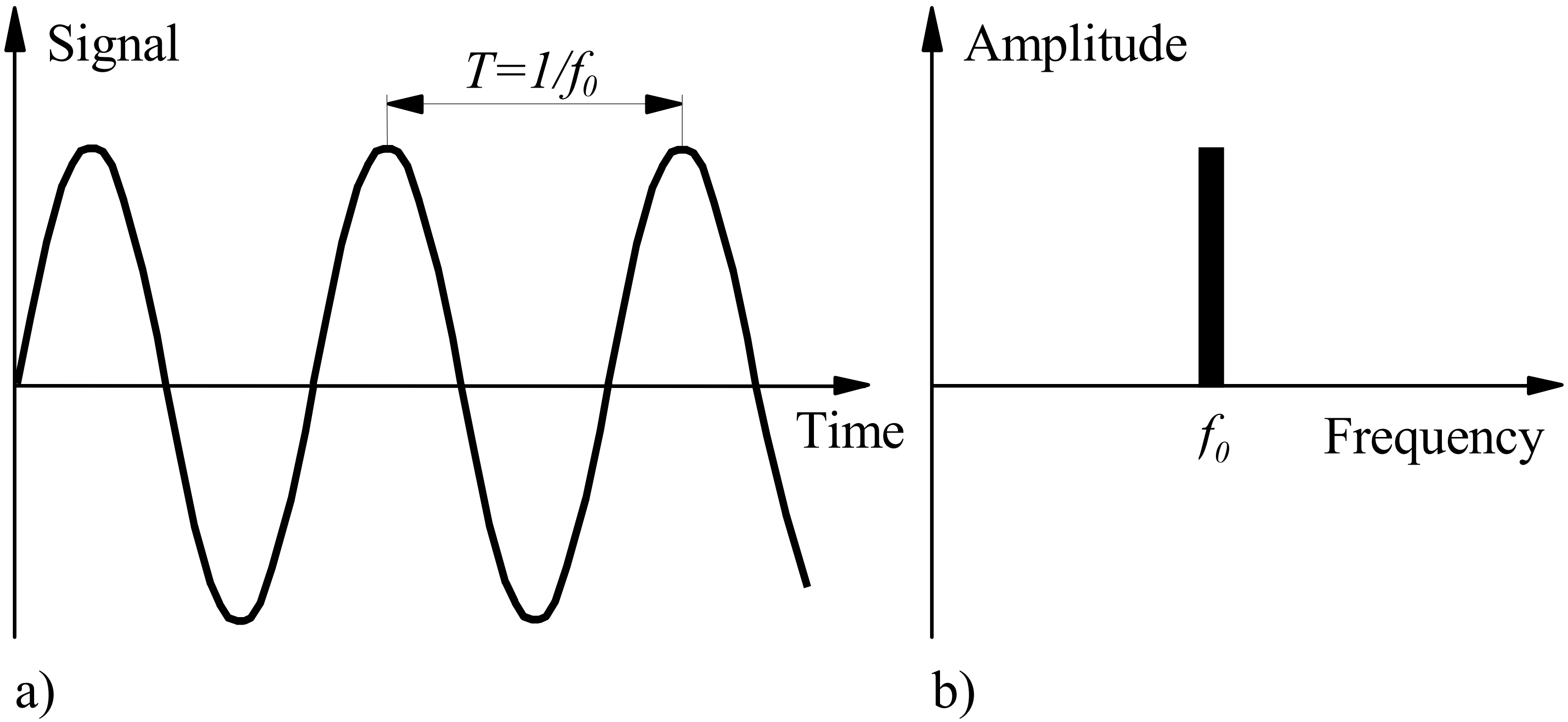
Any Periodic Signal can be considered a sum of a number of harmonic components

Periodic Signals are very common in Real life

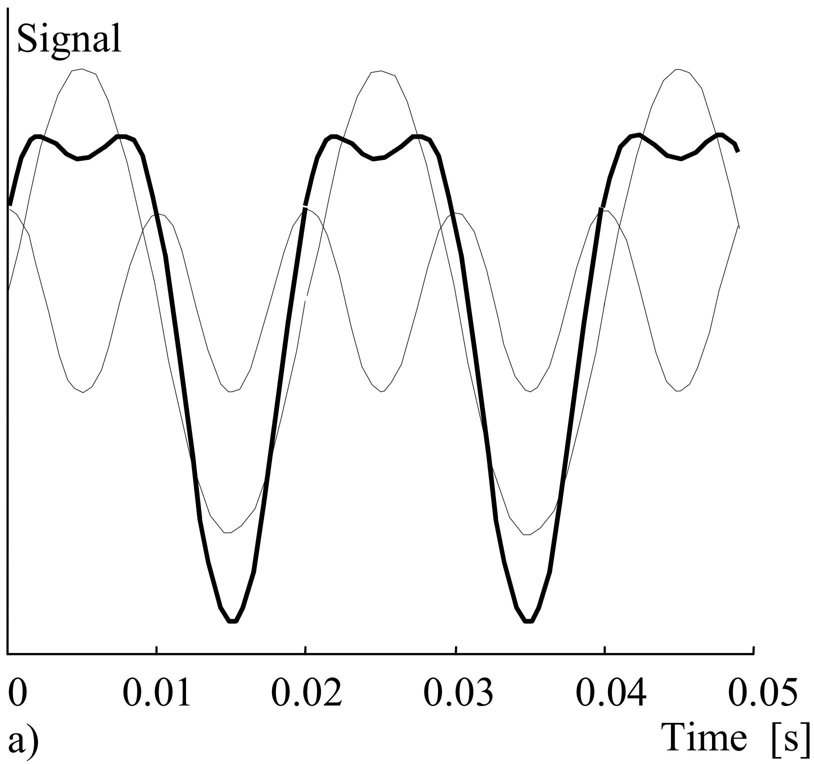
$$p(t) = p(t + nT)$$

$$p(t) = \sum_{n=1}^{N} \hat{p}_n \cos(2\pi\, n f_0 t + \varphi_n)$$

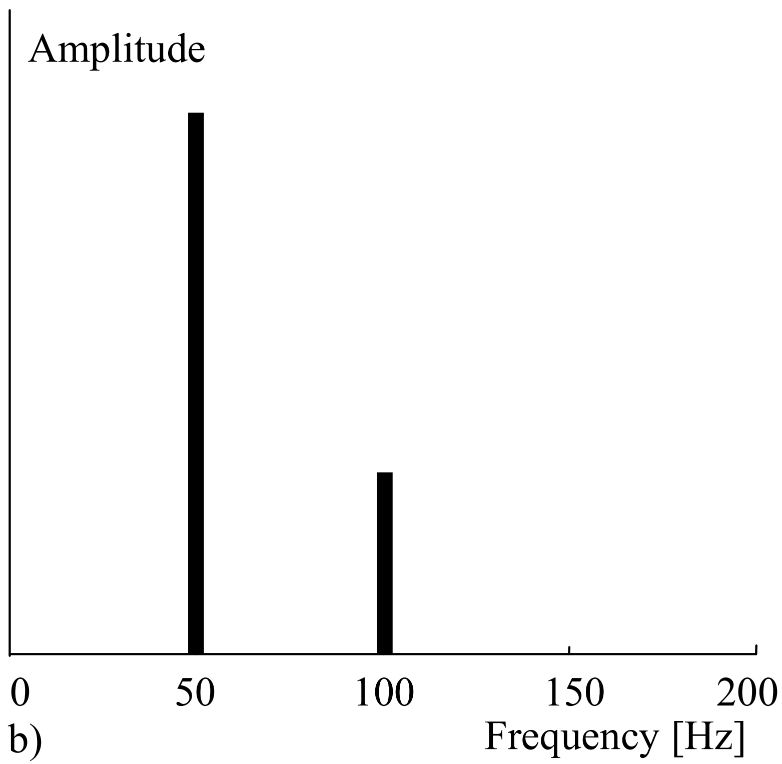

Amplitude
0
0
T/3
2T/3
T
Time [s]
1/T
2/T
3/T
4/T
Frequency [Hz]


Signal
$T=1/f_0$
Time
a)
Amplitude
f_0
Frequency
b)

Two Sine waves

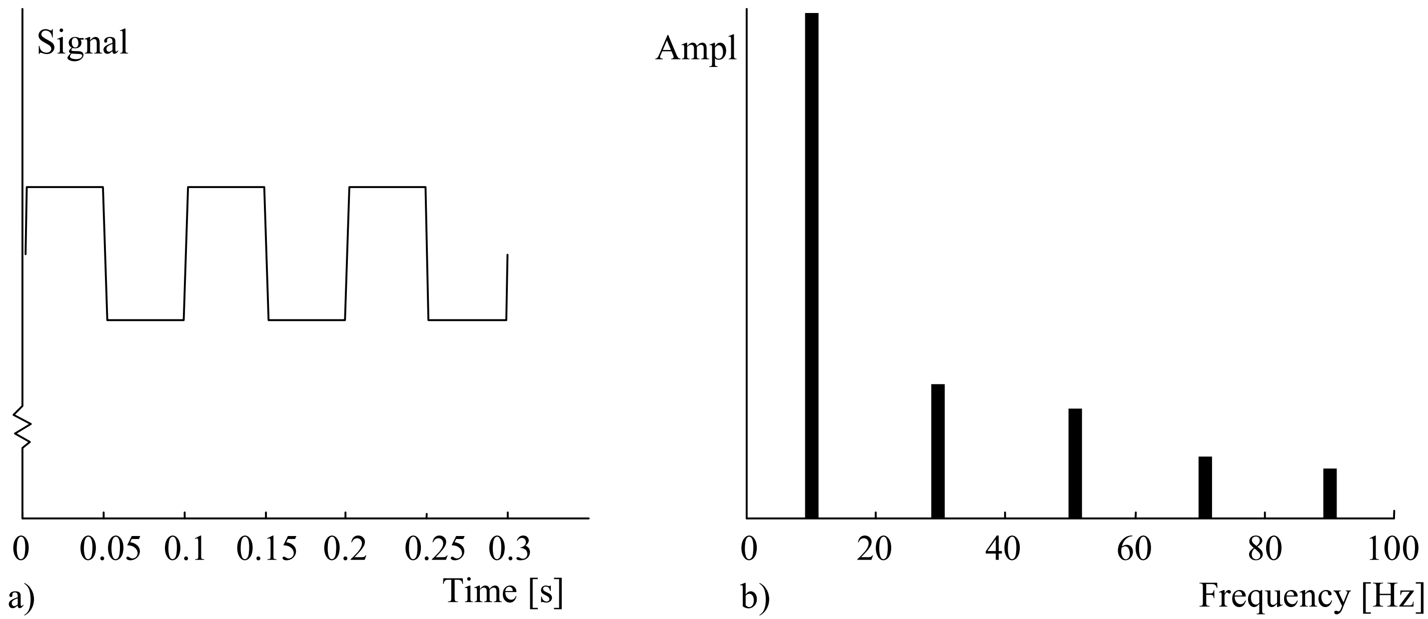


a)

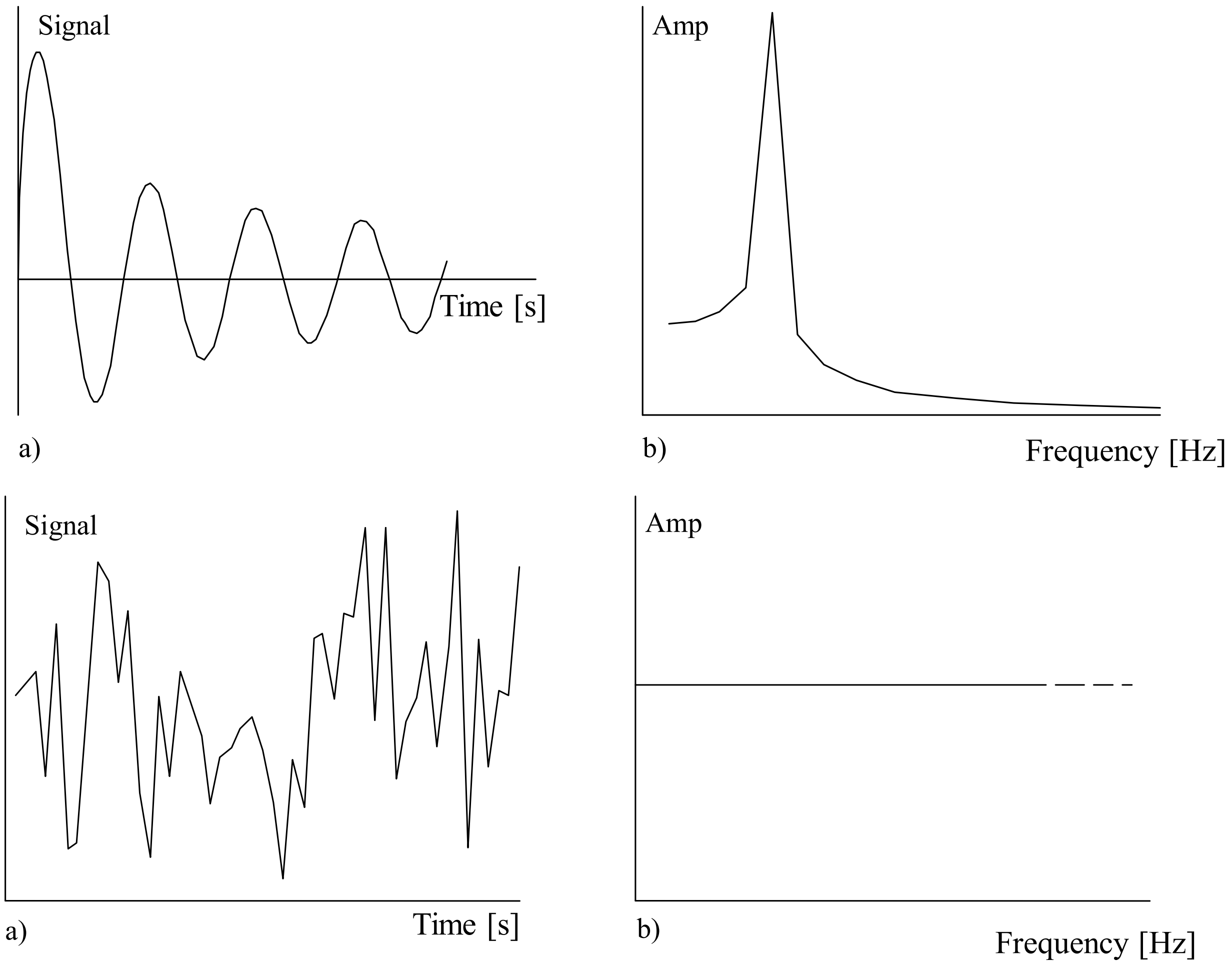


b)

Fourier Analysis



Fourier Analysis



Fourier Analysis

$$a(t) = \beta_0 + \sum_{n=1}^{\infty} \beta_n \cos(n\omega_o t) + \sum_{n=1}^{\infty} \gamma_n \sin(n\omega_o t)$$

$$\beta_0 = \int_{-T/2}^{T/2} a(t) 1 dt \Bigg/ \int_{-T/2}^{T/2} 1^2 dt = \frac{1}{T} \int_{-T/2}^{T/2} a(t) dt$$

$$\beta_n = \int_{-T/2}^{T/2} a(t) \cos(n\omega_0 t) dt \Bigg/ \int_{-T/2}^{T/2} \cos^2(n\omega_0 t) dt = \frac{2}{T} \int_{-T/2}^{T/2} a(t) \cos(n\omega_0 t) dt$$

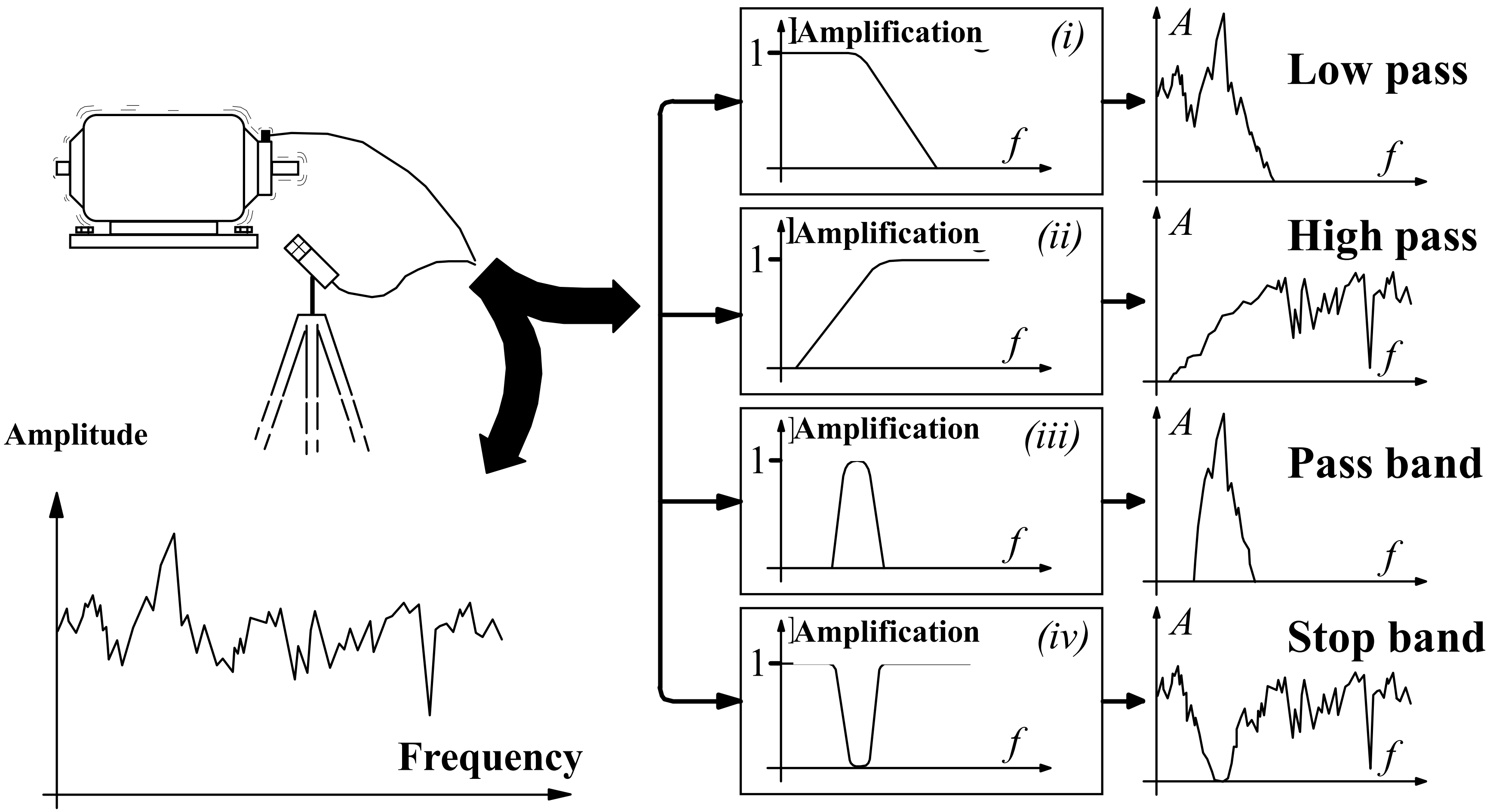
$$\gamma_n = \int_{-T/2}^{T/2} a(t) \sin(n\omega_0 t) dt \Bigg/ \int_{-T/2}^{T/2} \sin^2(n\omega_0 t) dt = \frac{2}{T} \int_{-T/2}^{T/2} a(t) \sin(n\omega_0 t) dt$$

Fast Fourier Transform (FFT)

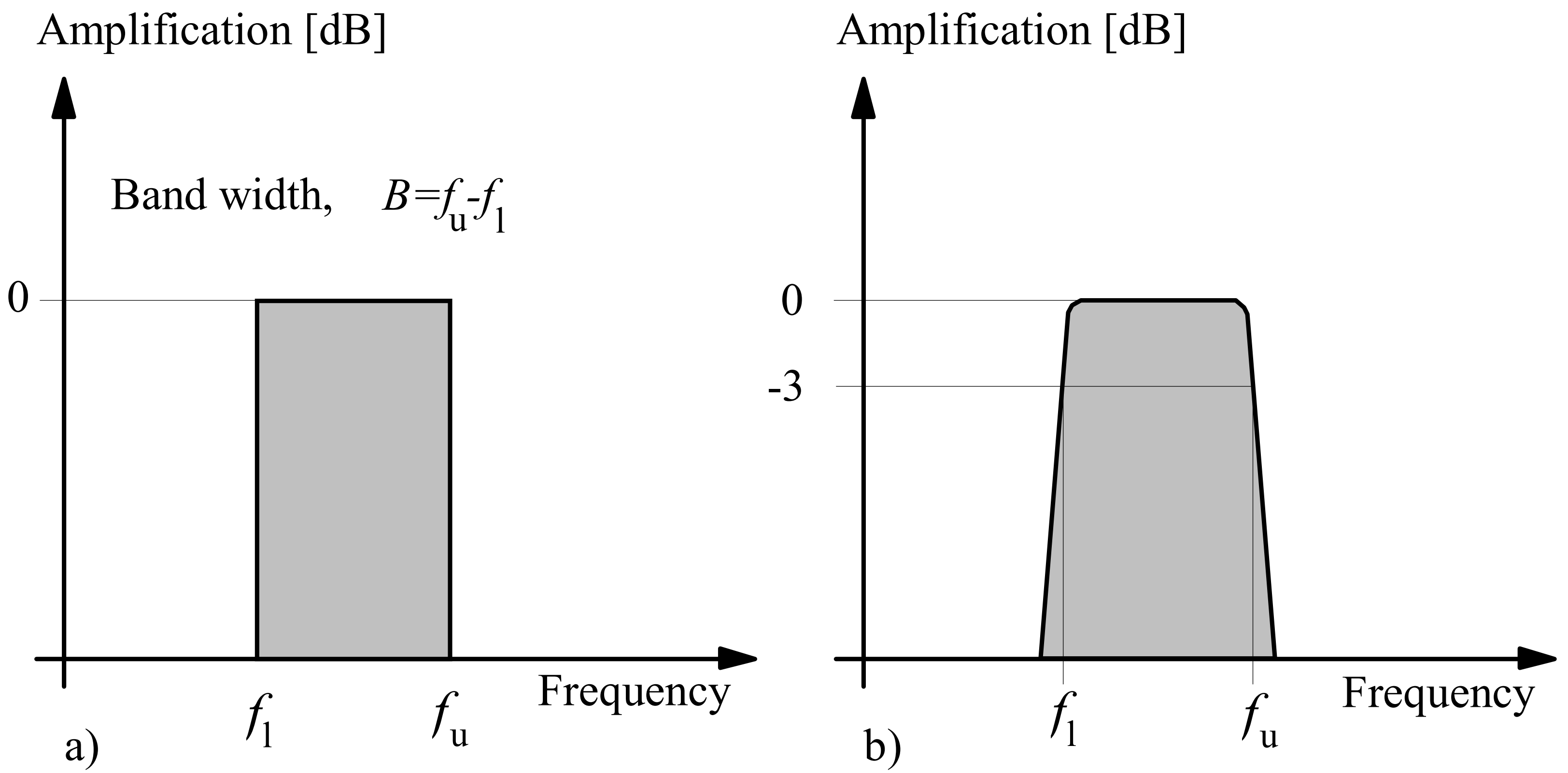
Assignment:
Read the MATLAB help about the fft function.
[put website!!]

Filters

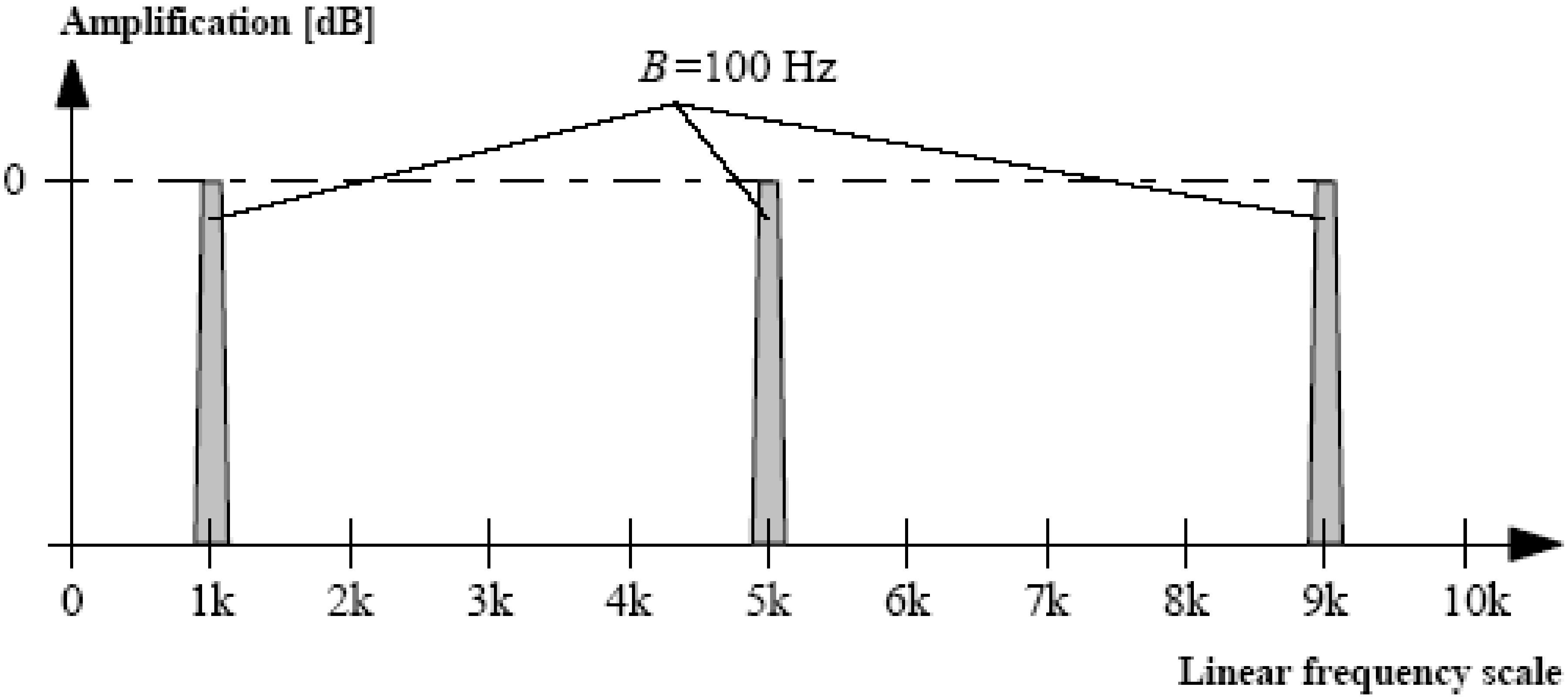
Types of Filters



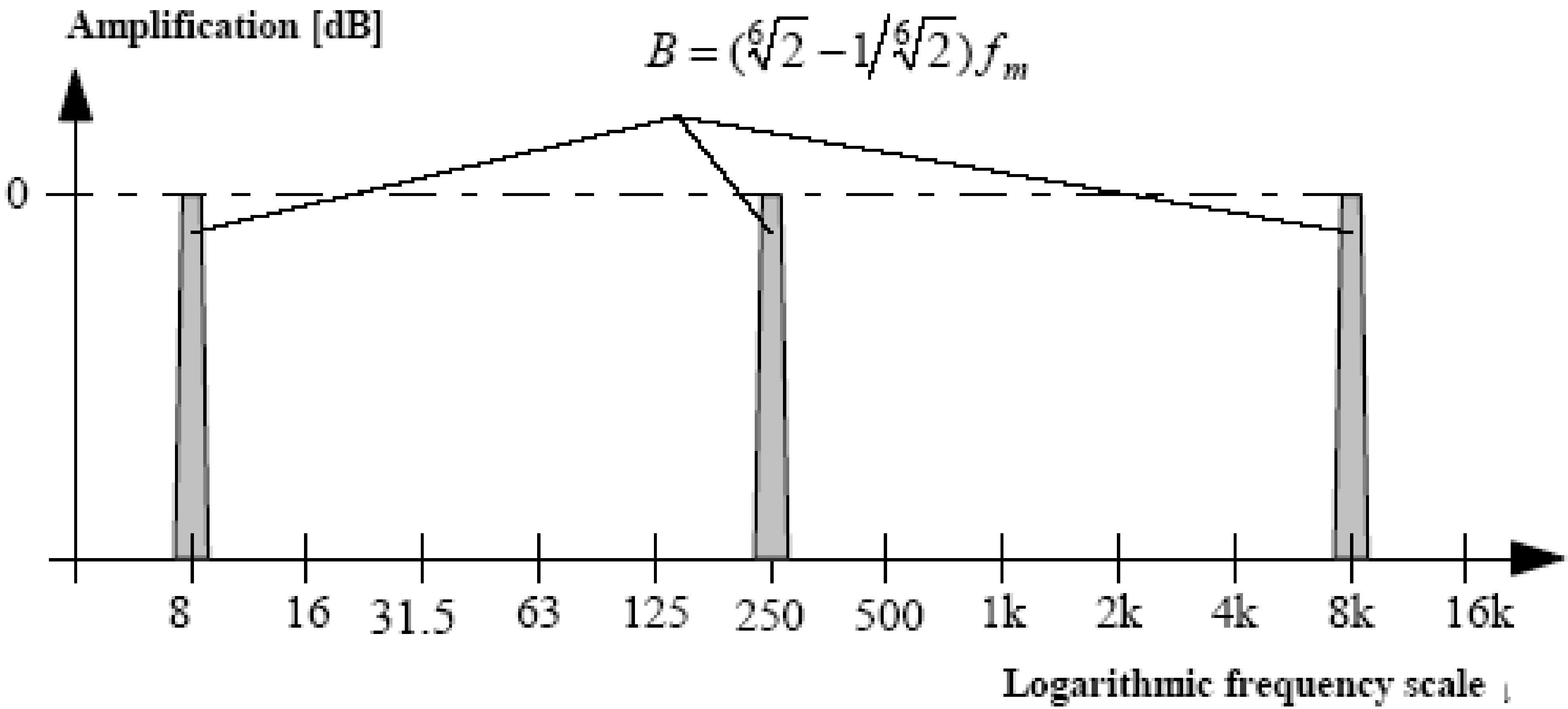
Band Pass Filters



CAB Filter


Amplification [dB]
B =100 Hz
0
0
1k
2k
3k
4k
5k
6k
7k
8k
9k
10k
Linear frequency scale

CRB Filter


Amplification [dB]
$B = (\sqrt[6]{2} - 1/\sqrt[6]{2}) f_m$
0
8
16
31.5
63
125
250
500
1k
2k
4k
8k
16k
Logarithmic frequency scale

Octave and Third Octave Filters

	Octave band filter	Third-octave band filter
Lower frequency limit	$f_l = f_c \big/ \sqrt{2}$	$f_l = f_c \big/ \sqrt[6]{2}$
Upper frequency limit	$f_u = \sqrt{2} f_c$	$f_u = \sqrt[6]{2} f_c$
Bandwidth	$B = f_u - f_l = \left(\sqrt{2} - 1\big/\sqrt{2}\right) f_c$	$B = f_u - f_l = \left(\sqrt[6]{2} - 1\big/\sqrt[6]{2}\right) f_c$
Center frequency	$f_c = \sqrt{f_l f_u}$	$f_c = \sqrt{f_l f_u}$

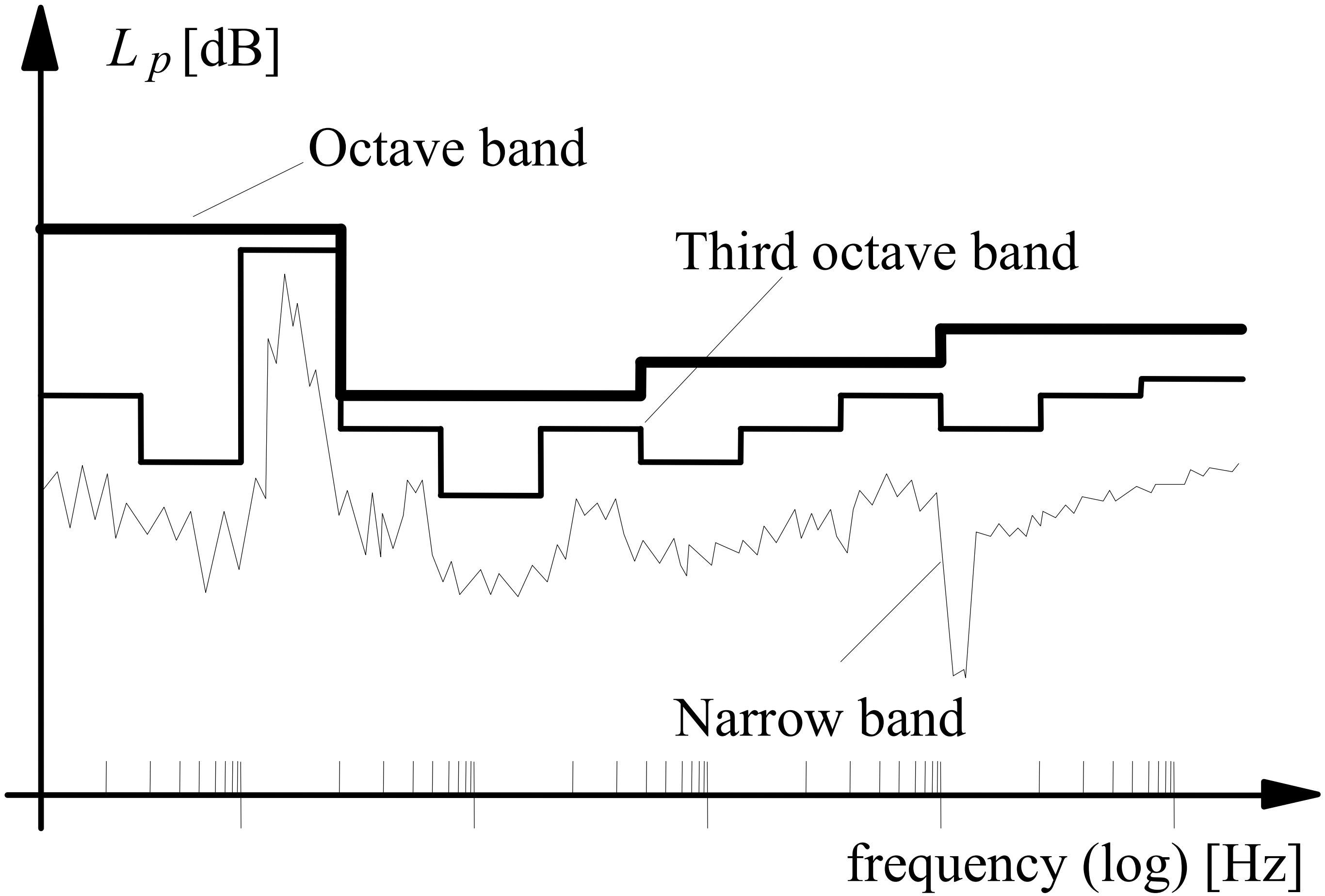
Band no.	Center frequency f_c [Hz]	3rd-octave band filter $f_l - f_u$ [Hz]	Octave band filter $f_l - f_u$ [Hz]	Band no.	Center frequency f_c [Hz]	3rd-octave band filter $f_l - f_u$ [Hz]	Octave band filter $f_l - f_u$ [Hz]
1	1.25	1.12 - 1.41		23	200	178 - 224	
2	1.6	1.41 - 1.78		24	**250**	224 - 282	**178 - 355**
3	**2**	1.78 - 2.24	**1.41 - 2.82**	25	315	282 - 355	
4	2.5	2.24 - 2.82		26	400	355 - 447	
5	3.15	2.82 - 3.55		27	**500**	447 - 562	**355 - 708**
6	**4**	3.55 - 4.47	**2.82 - 5.62**	28	630	562 - 708	
7	5	4.47 - 5.62		29	800	708 - 891	
8	6.3	5.62 - 7.08		30	**1000**	891 - 1120	**708 - 1410**
9	**8**	7.08 - 8.91	**5.62 - 11.2**	31	1250	1120 - 1410	
10	10	8.91 - 11.2		32	1600	1410 - 1780	
11	12.5	11.2 - 14.1		33	**2000**	1780 - 2240	**1410 - 2820**
12	**16**	14.1 - 17.8	**11.2 - 22.4**	34	2500	2240 - 2820	
13	20	17.8 - 22.4		35	3150	2820 - 3550	
14	25	22.4 - 28.2		36	**4000**	3550 - 4470	**2820 - 5620**
15	**31.5**	28.2 - 35.5	**22.4 - 44.7**	37	5000	4470 - 5620	
16	40	35.5 - 44.7		38	6300	5620 - 7080	
17	50	44.7 - 56.2		39	**8000**	7080 - 8910	**5620 - 11200**
18	**63**	56.2 - 70.8	**44.7 - 89.1**	40	10000	8910 - 11200	
19	80	70.8 - 89.1		41	12500	11200 - 14100	
20	100	89.1 - 112		42	**16000**	14100 - 17800	**11200 - 22400**
21	**125**	112 - 141	**89.1 - 178**	43	20000	17800 - 22400	
22	160	141 - 178					

Addition of Uncorrelated Sound Fields

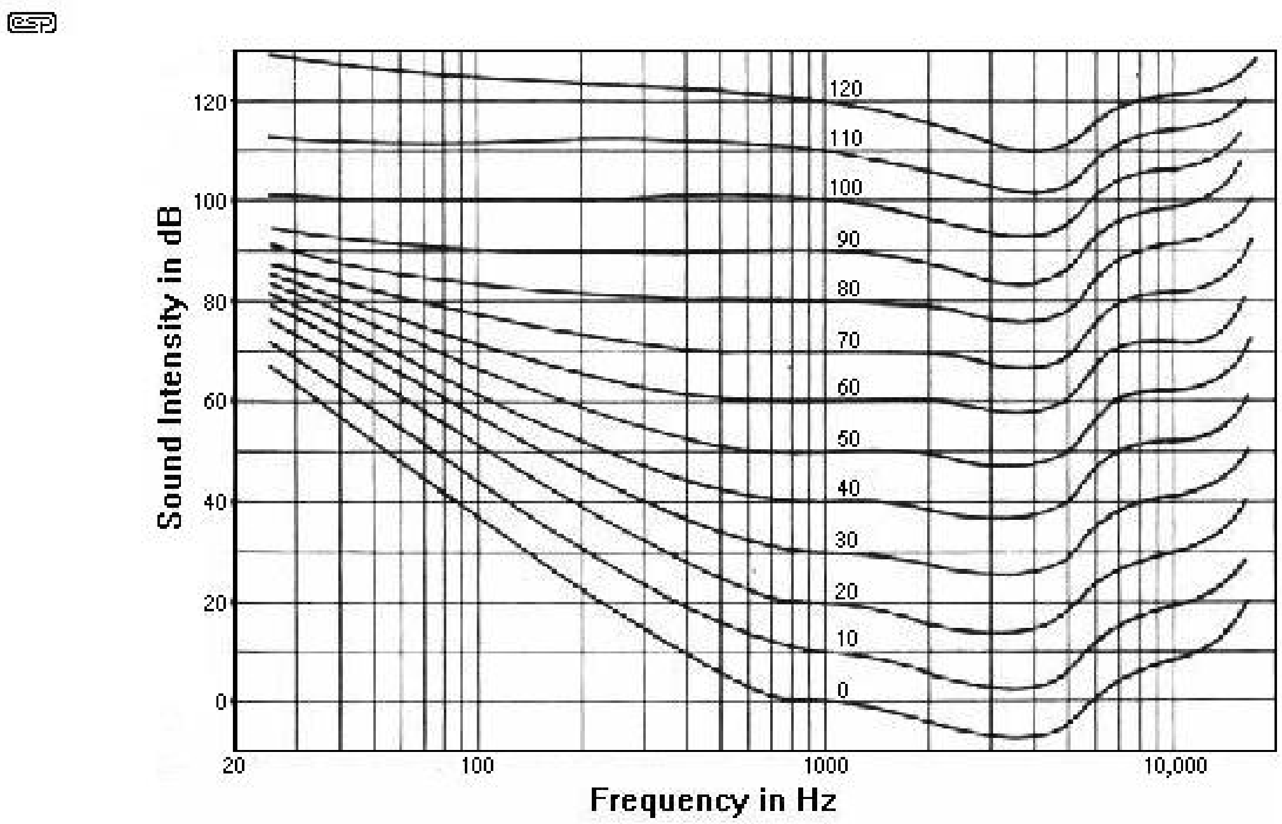
$$L_{p_{tot}} = 10 \cdot \log \sum_{n=1}^{N} 10^{L_{pn}/10}$$

Proof of this in sheet 1, problem 2

Narrow band vs. Octave bands

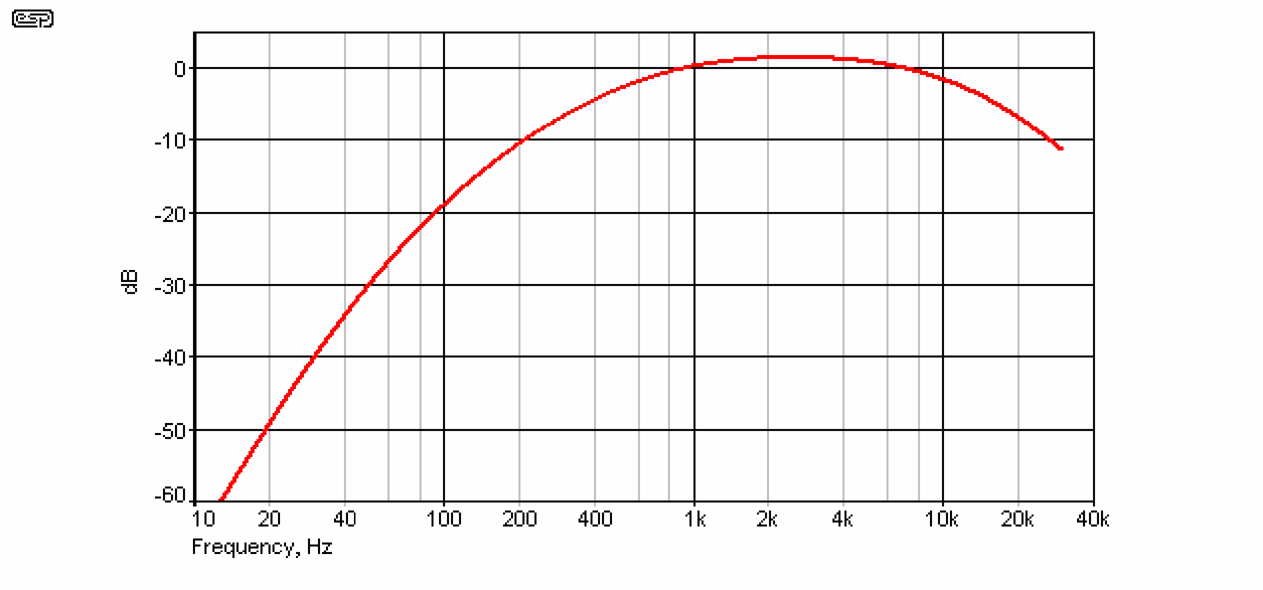


A-weighted SPL



Loudness Curves

A-weighted Sound Pressure Level



Freq (Hz)	Level (dB)
31.5	-39.4
63	-26.2
125	-16.1
250	-8.6
500	-3.2
1000	0
2000	1.2
4000	1
8000	-1.1
16000	-6.6